

# Stochastic Multicriteria Acceptability Analysis (SMAA)

theory, applications, and software

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# SMAA history



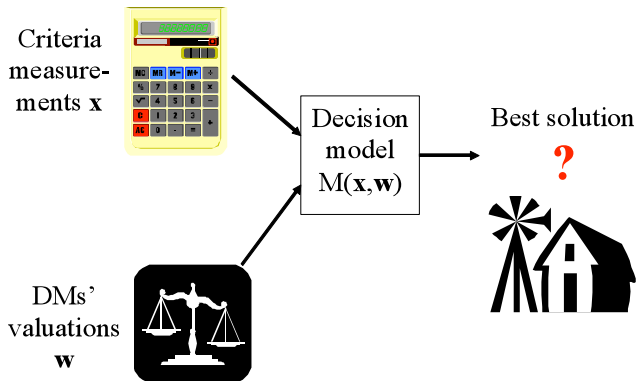
- In 1990, Helsinki decided that Vuosaari needed to be reserved for a general cargo harbour. In 1992 a new city plan was approved
- Environmental Impact Assessment (EIA) needed to be done
- EIA required valuations supporting each alternative to be described
- Politically very sensitive decision: DMs are not willing to provide preference information
- $\Rightarrow$  development of SMAA

# SMAA(-2)

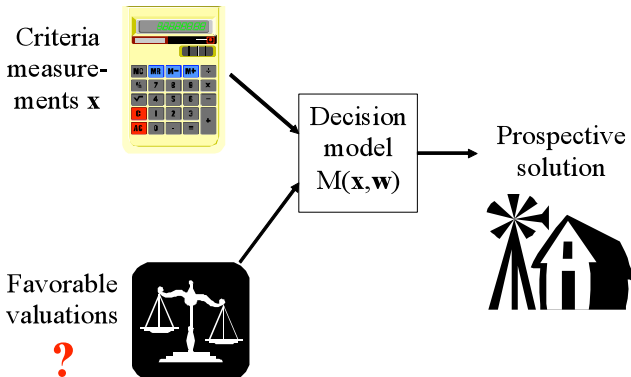
- SMAA(-2) is a multi-criteria decision aiding (MCDA) method for ranking a set of alternatives evaluated on basis of a set of criteria
- Preference information expressed with a weight vector and a value function of a commonly accepted shape (in practice often linear one)
- Uncertain or imprecise criteria values are represented by stochastic variables that map the deterministic value functions to value distributions

Lahdelma & Salminen, EJOR, 1998 / Tervonen & Figueira, JMCD, 2008

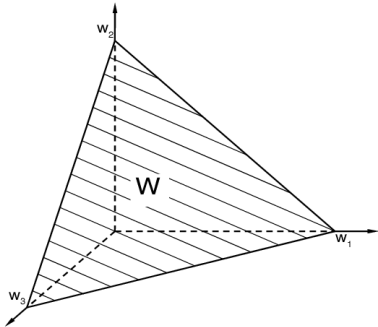
# Traditional MCDA



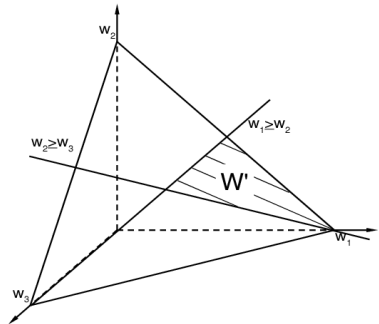
# Inverse approach



# Weight space



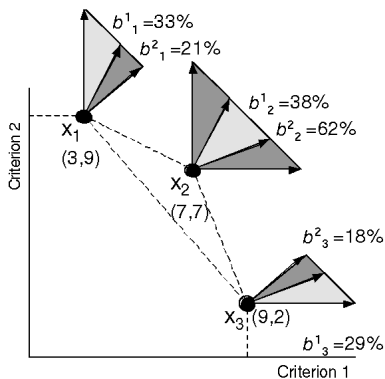
- Total lack of preference information is represented by a uniform joint probability distribution of the weight space



- If some preference information is available, the weight space can be restricted with linear constraints

## Rank acceptability index

Describes the share of different weights and criteria measurements ranking an alternative on a certain rank



$$b_i^r = \int_{\xi \in \mathcal{X}} f_{\mathcal{X}}(\xi) \int_{w \in W_i^r(\xi)} f_W(w) dw d\xi$$

# Central weight vector & confidence factor

- **Central weight vector** describes the preferences of a typical DM supporting this alternative with the assumed preference model
  - CW's are used for inverse approach: instead of asking preferences and giving results, answers the question "which preferences support an alternative to be the most preferred one?"
- **Confidence factor** is the probability for an alternative to be the preferred one with the preferences expressed by its central weight vector
  - CF measures whether the criteria measurements are accurate enough to discern the efficient alternatives

# Extensions

- SMAA-O for ordinal criteria that are implemented by simulating all piecewise linear value functions consistent with the ordinal preference information

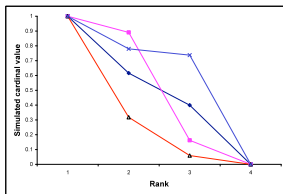
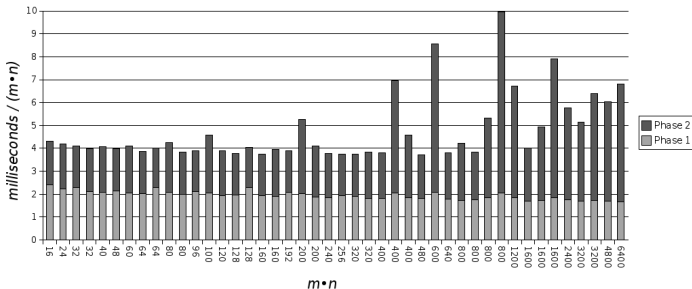


Figure: A sample ordinal-to-cardinal mapping of SMAA-O

- Cross confidence factors for discriminating among very imprecise alternatives
- SMAA-3, SMAA-TRI, SMAA-III, SMAA-D, SMAA-A, SMAA-P, SMAA-CEA, ...

# Computation

- Analytical techniques based on discretizing the integrals with respect to each dimension are infeasible, so the integrals are estimated through Monte Carlo simulation
- 10000 simulations provide sufficient accuracy for the indices
- Algorithm has less-than squared mean complexity and is very fast in practice



# Application: Locating a university kindergarten in Madrid

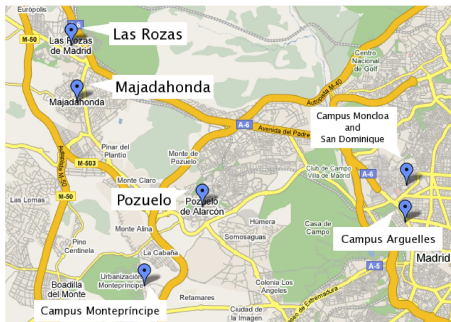


Figure: Alternative locations

Tervonen & al., Springer, 2010

- San Pablo CEU needed to build a kindergarten for staff children (in 90's). In 2007, the process was re-initiated as a two-phase decision process for site selection.
- SMAA-III was applied for ranking 7 alternate locations based on both ordinal and cardinal criteria.

## Application: drug benefit-risk analysis

- Hansen & al. (Ann Intern Med, 2005) assessed safety and efficacy of four second generation antidepressants and concluded that there are “no significant differences among the drugs”
- In general, the assessment of antidepressants is hard; placebo effect is always present causing high uncertainty on the results
- Q's:
  - ① How can the benefit-risk (efficacy vs safety) assessment of second-generation antidepressant be structured based on evidence from the clinical trials?
  - ② Can we come up with something better than “no significant differences”?

# Data from meta-analysis

Study, Year (Reference)

Bennie et al., 1995 (33)\*

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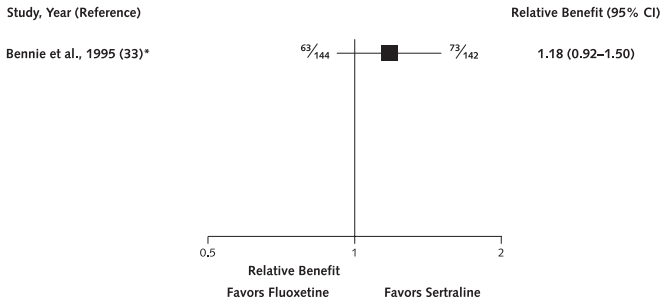
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Fluoxetine

Sertraline

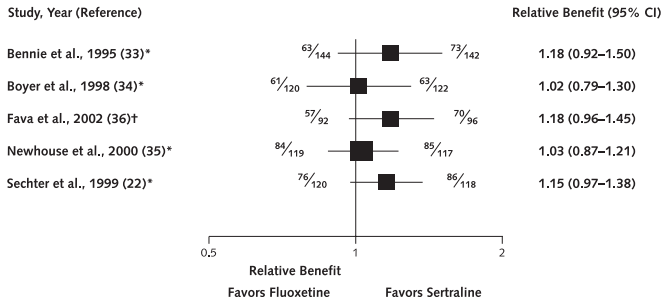
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# Data from meta-analysis



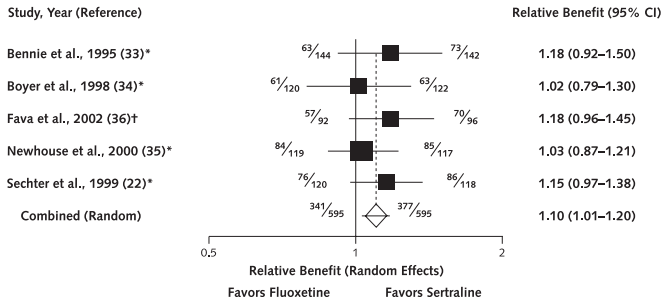
Hansen & al., Ann Intern Med, 2005

# Data from meta-analysis



Hansen & al., Ann Intern Med, 2005

# Data from meta-analysis



Hansen & al., Ann Intern Med, 2005

Problem formulation in MCDA terms:

- $m$  alternative treatments are evaluated with respect to efficacy and  $n - 1$  most important adverse drug reactions (ADRs)
- criteria measurements for efficacy are risk ratios (log-normal distributed) compared against Fluoxetine:

| Treatment   | Mean | 95% CI        |
|-------------|------|---------------|
| Fluoxetine  | 1.00 | (1.00 - 1.00) |
| Paroxetine  | 1.09 | (0.97 - 1.21) |
| Sertraline  | 1.10 | (1.01 - 1.20) |
| Venlafaxine | 1.12 | (1.02 - 1.23) |

- measurements for ADR criteria are normal distributed incidence rates

Tervonen & al., SOM Res Rep, 2010 (submitted to Stat in Med)

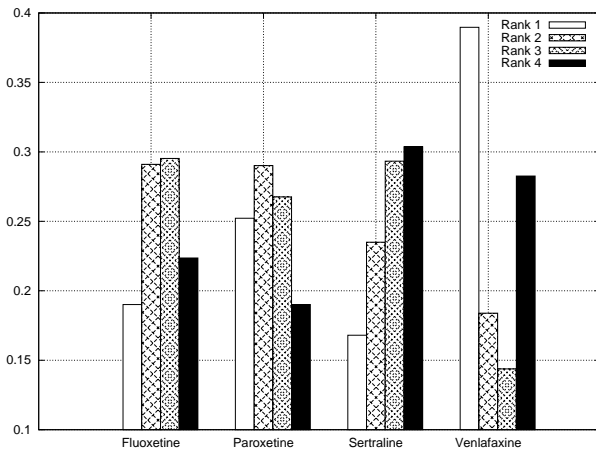
# Preference information

- We considered 3 scenarios:
  - ① Health policy decision making with no preferences
  - ② Prescription for mild depression
  - ③ Prescription for severe depression
- Ordinal swing weighting for prescription decisions

Table: Criteria scales

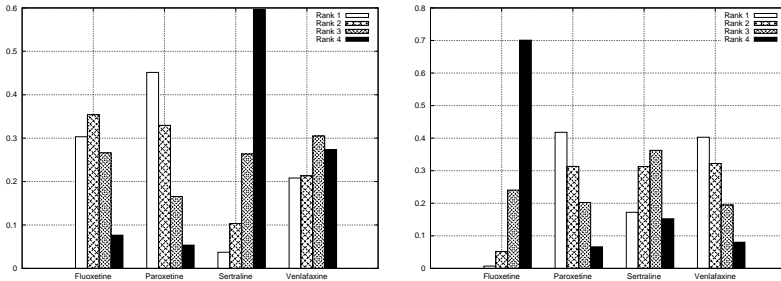
| Criterion      | Scale range  |
|----------------|--------------|
| Efficacy       | [0.98, 1.23] |
| Diarrhea ADRs  | [1, 20.6]    |
| Dizziness ADRs | [4.4, 24.4]  |
| Headache ADRs  | [8, 31.3]    |
| Insomnia ADRs  | [3.4, 21.3]  |
| Nausea ADRs    | [11.1, 34]   |

# Results (1)



**Figure:** Rank acceptability indices for the model without preference information.

## Results (2)



**Figure:** Rank acceptability indices from the scenarios of mild and severe depression.

# Software: JSMAA

## Main features

- Implements SMAA- $\{2,0,TRI\}$
- Save/load model in XML
- Results visualization

<http://smaa.fi>

The screenshot shows the JSMAA software interface. The left pane displays a tree view of the model structure:

- Benefit-Risk model of second order
  - Alternatives
    - Fluoxetine
    - Paroxetine
    - Sertraline
    - Venlafaxine
  - Criteria
    - Efficacy (selected)
    - Diarrhea
    - Dizziness
    - Headache
    - Insomnia
    - Nausea
  - Preferences
  - Results
    - RankAcc
    - CW

The right pane shows the configuration for the selected 'Efficacy' criterion:

- Criterion: Efficacy
- Name: Efficacy
- Type: Cardinal
- Scale: [0.98 - 1.23]
- Ascending:
- Measurements:
 

|             |           |       |   |       |
|-------------|-----------|-------|---|-------|
| Fluoxetine  | LogNormal | 0.0   | ± | 0.0   |
| Paroxetine  | LogNormal | 0.086 | ± | 0.056 |
| Sertraline  | LogNormal | 0.095 | ± | 0.044 |
| Venlafaxine | LogNormal | 0.113 | ± | 0.048 |

At the bottom of the window, a status bar indicates "Simulation complete."

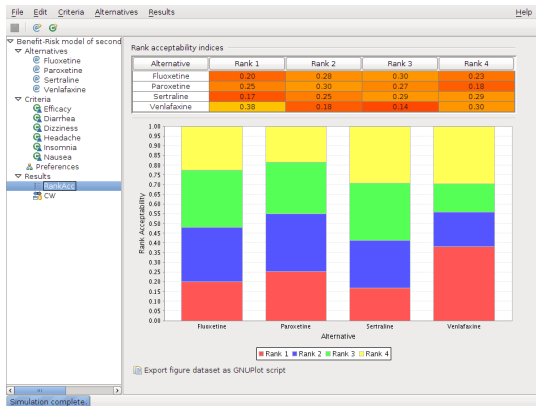
Tervonen, URPDM'2010

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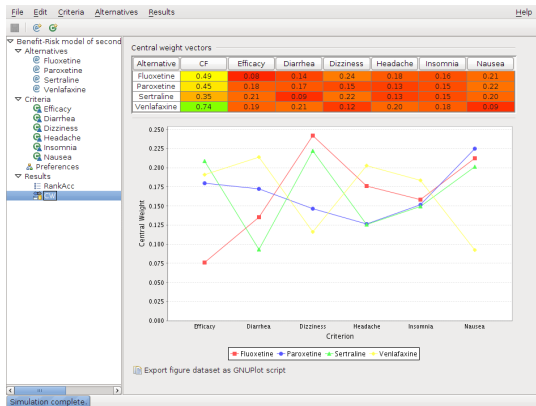
Tervonen, URPDM'2010

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Tervonen, URPDM'2010

# Conclusions

- Flexibility and maturity of the SMAA methodology allows “out of box” thinking and application of MCDA in new disciplines – most of real-life decision problems are multi-criteria by nature
- SMAA methods have been successfully applied in various real-life MCDA problems with imprecise criteria measurements and partial/missing preference information
- Cross-platform, open source implementation of SMAA methods is available from [www.smaa.fi](http://www.smaa.fi)



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Gracias por su atención – Obrigado pela sua atenção